Geometric Data $\rightarrow$ Structure

Grids –

- Closest pair
- Clustering
- Misc

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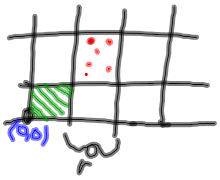
Grid

$r > 0$

G_r



$p = (x, y) \rightarrow id(p) = \left(\lfloor \frac{x}{r} \rfloor, \lfloor \frac{y}{r} \rfloor\right)$

$P \rightarrow G_r(P)$

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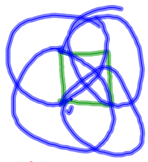
Closest Pair

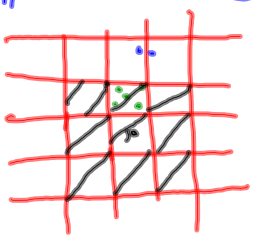
P = n points in the plane



Decision Version / Verification

$$r = CP(P) = \min_{\substack{p,q \in P \\ p \neq q}} \|p-q\|$$

$G_r(P)$



Linear time

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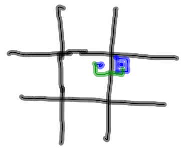
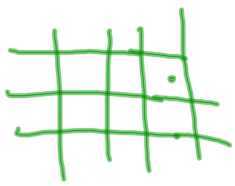


$r_i = CP(\langle p_1, p_2, \dots, p_i \rangle)$

p_{i+1} $\quad r_i = r_{i+1} \rightarrow O(1)$

$r_i > r_{i+1} \rightarrow G_{r_{i+1}}(P_{i+1})$

$O(i)$



k rebuilds $\Rightarrow O(nk)$

Rebuilds ?

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Lemma

$p_1, p_2, \dots, p_n$ – random permutation

$r_2, r_3, \dots, r_n$

k = # of i's s.t. $r_i \neq r_{i+1}$

$E[k] = O(\log n)$

Proof:

$Pr[r_i \neq r_{i+1}] \leq \frac{2}{i}$

$E[k] = \sum_{i=3}^{n} E[X_i] = \sum_i \frac{2}{i} = O(\log n)$ ∎

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Lemma

RT of CP algorithm is $O(n)$.

Proof

$X_i = 1$ if $r_i < r_{i-1} \Rightarrow O(i)$

$X_i = 0$ if $r_i = r_{i-1} \Rightarrow O(1)$

$RT = \sum_i (1 + X_i \cdot i)$

$E[RT] = \sum \left(1 + \underbrace{E[X_i]}_{\leq 2/i} \cdot i\right) = \sum_i 1 = O(n)$ ∎

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Lemma

P – set of n points

k – param

Compute in $O(n)$ time a disk of radius r covering k points of P s.t.

$r \leq 2 r_{opt}(P, k)$

$k=2$

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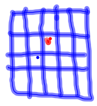
Geometric hashing

T – space of transformations

$[0,1]^6$



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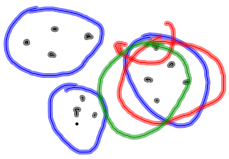
Covering by unit disks

P – set of points

Q: Find min # of unit disks covering P



$O(n^2)$

~

k disks suffice $O\binom{n^2}{k} \cdot O(nk) = n^{O(k)} \Rightarrow n^{O(k)}$

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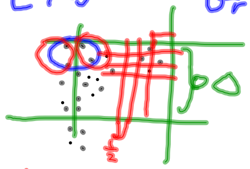
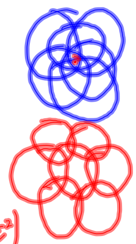
δ ← Parameter $\quad \delta = O(1/\varepsilon)$

$x \in [0, \delta]^2 \quad G_r + x$





$k = O(\delta^2) = O(\frac{1}{\varepsilon^2}) \Rightarrow n^{O(1/\varepsilon^2)}$

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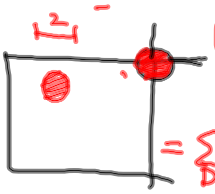
Claim

$E[\#\text{disks output}] \leq (1+\varepsilon)\,opt$

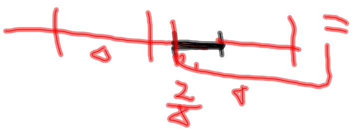
RT $n \cdot n^{O(\delta^2)} = n^{O(1/\varepsilon^2)}$

Proof

$P_i[\text{disk is split}] \leq \frac{4}{\delta}$



$E[\text{size of approx sol}] \leq E[\text{opt partition on grid}] = \sum_{D \in opt} E[1+4X_D] = opt(1+\frac{16}{\delta})$



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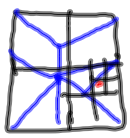
Quadtrees

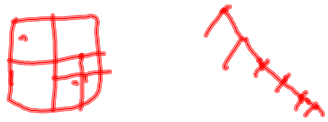
G_r – is a canonical grid if r is a power of 2.

$P \subseteq [0,1]^2$



list of intersecting triangles

P – set of points



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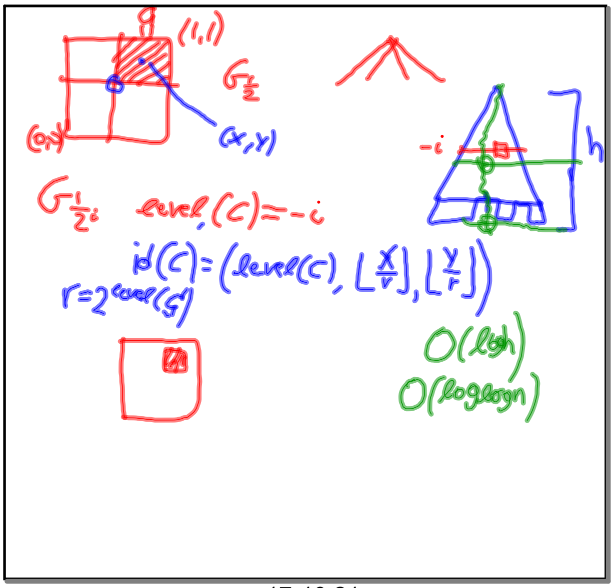
(1,1)
G 1/2
(0,0)
(x,y)
-i
h
G 1/2^i
level(C) = -i
id(C) = (level(C), ⌊x/r⌋, ⌊y/r⌋)
r = 2^level(C)
O(log h)
O(log log n)

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